

Minnesota State High School
Mathematics League
Individual Event

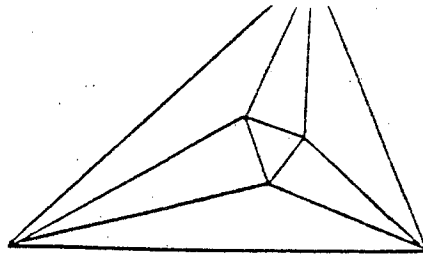
3A 1991-

Instructions: The first question should require very little time or computation. It is worth 1 point. Each of the other questions is worth 2 points. Put your answer to each question on the indicated blank on this page.

- _____ 1. If $f(x) = \frac{1 - \frac{1}{x}}{x}$, what is $f(2/3)$?
- _____ 2. Suppose it is known that y varies inversely as the square of x . If $y = \frac{2}{3}$ when $x = \frac{1}{2}$, what will y equal when $x = \frac{2}{3}$?
- _____ 3. Given that $a + b = 3$ and that $a^3 + b^3 = 13$, what is ab ?
- _____ 4. Let $f(x) = \frac{1}{x} - \frac{1}{x+1}$. Then $f(\frac{1}{x}) + f(x) = \frac{1}{x} + p(x)$ where $p(x)$ is a polynomial. Find $p(x)$.

Name _____

School _____



Minnesota State High
Mathematics I
Individual

3A 1992-

Instructions: The first question should require very little time or computation. It is worth 1 point. Each of the other questions is worth 2 points. Put your answer to each question on the indicated blank on this page.

_____ 1. If $f(x) = \frac{x-1}{x+1}$, what is $f\left(\frac{3}{4}\right)$?

_____ 2. The function f of question 1 is expressed as the quotient of two first degree polynomials. Express as the quotient of two first degree polynomials the expression
$$g(x) = f(x^{-1}) + f(x) + [f(x)]^{-1}.$$

_____ 3. If $f(x) = \frac{x}{4^0} + \frac{x^3}{4^1} + \frac{x^5}{4^2} + \frac{x^7}{4^3} + \dots + \frac{x^{21}}{4^{10}}$, find $f(2)$

_____ 4. If you are allowed to pick real numbers p , q , and r in any way that you please, subject only to the condition that they all be non-negative, what is the smallest value possible for
$$\frac{(p^2+p+1)(q^2+q+1)(r^2+r+1)}{pqr}?$$

Name _____

School _____

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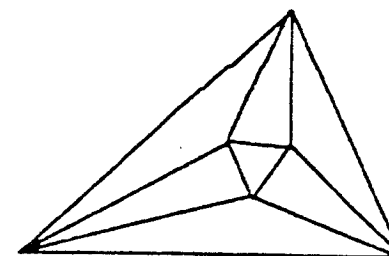
3A 1991-

Each question should require very little time or computation. It is worth 1 point. Each question is worth 2 points. Put your answer to each question on the indicated blank on this page.

1. If y varies inversely as the square of x . If $y = \frac{2}{3}$ when $x = \frac{1}{2}$, find y when $x = \frac{2}{3}$.

2. If $a^3 + b^3 = 13$, what is ab ?

3. If $f\left(\frac{1}{x}\right) + f(x) = \frac{1}{x} + p(x)$ where $p(x)$ is a polynomial. Find $p(x)$.



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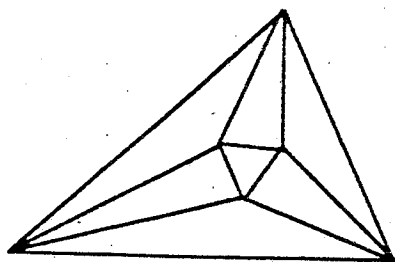
1994-95 Event 3A

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. List all solutions (and only solutions, of course) to $\sqrt{x+4} = 2-x$.
2. Express $\frac{(a+1)+2(a-1)^{-1}}{1+(a-1)^{-1}}$ as a sum of two terms, each involving nothing more complicated than a power of a .
3. In an increasing geometric sequence, the product of the first three terms is 1, and their sum is $\frac{7}{2}$. What is the fifth term?
4. The expansion of $\left(x^2 + \frac{1}{3\sqrt{x}}\right)^{12}$ includes a term of the form cx^4 where c is a rational number. Express c as the quotient of two relatively prime integers.

Name

Team



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1995-96 Event 3A

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

_____ 1. Find x , rounded accurate to two places beyond the decimal, if $\sqrt{1.79 + \sqrt{x}} = 1.7$.

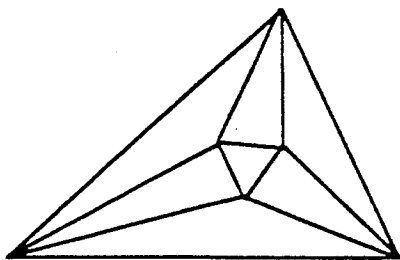
_____ 2. Given that $f(x) = \frac{3x}{x+1}$, find $f\left(f\left(\frac{1}{2}\right)\right) - \left(f\left(\frac{1}{2}\right)\right)^2$.

_____ 3. Given $f(x) = \frac{3x}{x+1}$, find a function $g(x)$ such that $g(f(x)) = x$ for every x .

_____ 4. If $\frac{3x-y}{x+y} = \frac{1}{2}$, what is the numerical value of $\frac{3x+y}{y-x}$?

Name

Team



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1996-97 Event 3A

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

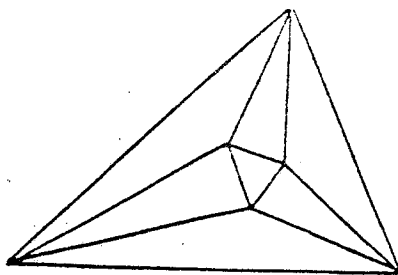
_____ 1. Express $4^{\frac{-3}{2}} + 4^{-1} + 4^{\frac{-1}{2}} + 4^0 + 4^{\frac{1}{2}} + 4^1 + 4^{\frac{3}{2}}$ as the quotient of two relatively prime integers.

2. Express $\frac{1}{x+1} + \frac{1}{2x^2-x-3} - \frac{2}{2x+3}$ as the quotient of two polynomials.

_____ 3. What ordered pair of real numbers (x,y) will give the largest possible value for $\frac{4-3y-y^2}{1+x+x^2}$?

_____ 4. The expressions on the left and right sides of $\frac{b^3}{a^3} = \frac{3a^2-5ab+3b^2}{5a^2-3ab+5b^2}$ are said to be symmetric in the real variables a and b . Such expressions are often explored by making the substitution $b = ta$. In this case, this substitution leads to a polynomial equation of the form $P(t) = 0$. Find $P(t)$

Name _____ Team _____



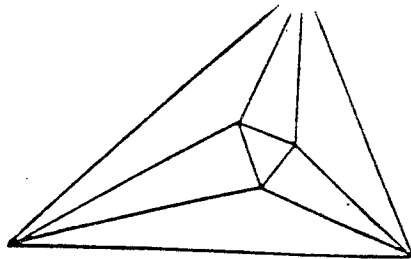
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1997-98 Event 3A

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

- _____ 1. Suppose that $f(x) = x^2 - \frac{12}{x+1}$. Find a real number x so that $f(2x) = 0$.
- _____ 2. Suppose that $a - b = 3/4$ and $a^2 - b^2 = 5/7$. Find the value of a and write it as a rational number in reduced form.
- _____ 3. Given that $a^{2z} = 5$, express $\frac{a^{3z} - a^{-3z}}{a^z - a^{-z}}$ as a rational number in reduced form.
- _____ 4. Factor $x^4 + 6x^2 + 25$ into a product of two quadratics with integer coefficients.
- _____

School _____



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1998-99 Event 3A

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. Simplify $\frac{x^{-1} + x^{-2}}{x^{-3}}$ and write it as a simple expression containing only positive exponents.

2. Find rational numbers a and b in lowest terms so that

$$\frac{\sqrt{x^2 y^3} \sqrt[4]{x^3 y^2}}{\sqrt[3]{x} \sqrt{y}} = x^a y^b.$$

$a =$ _____, $b =$ _____ . (One point for each correct answer.)

- _____ 3. The shoe size of a normal man varies approximately as the $3/2$ -power of his height. If the average 6-foot man wears a size 11 shoe, what would you predict as the shoe size of a 7-foot man? (Give your answer to the nearest two places after the decimal point.)

- _____ 4. Suppose f is a function that satisfies $\frac{1}{x}f(-x) + f\left(\frac{1}{x}\right) = 2x$ for all x different from 0. What is the value of $f(2)$?

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